

# PASSIVITY BASED CONTROL OF A 3-DOF ROBOT MANIPULATOR

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**Abstract**—Robot manipulator are being applied in factory automation as well as in many different applications. These systems are considered to be highly non linear. Several results for rigid robot manipulators control were obtained by different methods. We can state for instance the adaptive motion control, robust control, intelligent control. In many cases these methods have been shown to be globally convergent. There are a vast array of design techniques for generating control strategies, where the model of the system is known. Some of these techniques have been successfully used. our emphasis in this paper is on design technique that is applicable when the system is assumed to be passive.

**Keywords:** Robot Manipulator, Adaptive control, passivity

## 1. INTRODUCTION

Robotics appears like a discipline permitting the implementation of the means to prolong or to transpose the intelligent motor actions of the man. In a first aimed objective, it consists to find a method to replace the man in some domains judged dangerous or harmful.

The robotics became a fully-fledged discipline, it regroupes electronics, electrotechnics, mechanics and computer data processing. Of this multidisciplinary are born a vocabulary and a very specific vision of the problems connected to the robots. These problems are at a time the conception of new mechanical structures, the development of new control laws, and more important performance demand. Robots manipulator control constitutes one of the main préoccupations of the research at the present time. This interest is bound closely to the difficulty to control a robot to which one asks precision and a complicated tasks execution. this precision is more difficult to get if the speed of evolution increases. One of the first tentatives, in the control of robots, consist in simplifying the dynamic equations of the model in order to be able to apply the methods of classical control (PID). But this solution based on stationary gains control laws loses its performances as soon as the robot is brought to evolve to big speeds (the terms of coupling increase with the speed). The second method takes into account the complete model of the robot. This technique, known as dynamic control, has the tendency to compensate the undesirable terms and defines the dynamics

of evolution of the robot manipulator in closed loop with a linear controller. However this compensation is only efficient if the dynamic parameters are identified with precision. On the other hand, some terms as friction and carried load, are difficult to determine directly because they vary during the time.

To come up with solutions to these difficulties of identification, the research moved on robust approaches. The notion of robustness is taken differently by the researchers working in the field of robust control. By robustness, one has the tendency to think about performance in the more part of the cases, but it is necessary to keep in mind that the main characteristic of a system control is its stability. Then, when this condition is respected one tries to get performances as interesting as possible for a given application. One can say that the robustness means that a certain level of performance is assured in spite of uncertainties on the process that is controlled in relation to its nominal model. Adaptive control gained attention since they use algorithms of adaptation for the parameter estimation of the robot model. Many apparently different approaches to adaptive control have been proposed in the litterature.

These solutions appear like the most suitable, insofar as even though an error appears. In this context, we investigate control scheme that uses stability theorem based on passivity naturally associated with power dissipation. Such a concept can be defined for linear as well as non linear systems.

In this paper we want to show some notes of the adaptation algorithm for adaptive control of robot manipulator where the passivity property is used. our aim is to apply this technique to a three-degree-of-freedom robot manipulator and investigate the global properties of the designed controller.

Mathematical models of mechanical systems are usually described by the Hamiltonian or Euler-Lagrange equations. These models have a structure that makes them very attractive for the design of control algorithms based on energy or passivity consideration. This approach has already proved to be effective in the control design for mechanical systems. Several results based on this approach have been proposed to obtain near optimal controllers [1], [2], [3], [4], [5]. One of the

control strategies we found in the literature is the velocity field control (VFC) which describes the control of a robot to follow a contour. The desired contour is described by a velocity tangent vector [6]. In this optics, we try to apply the control strategy based on passivity to a 3-dof robot manipulator built in LRV laboratory. This robot was modeled by its kinematic and dynamic equations reflecting its natural and true behavior. In fact, this is a start of a work being developed for non linear teleoperator system that target coordination of the master and slave manipulators as well as passivity of the overall system.

This paper is organized as follows. Section II presents the dynamics of the robot manipulator, Section III presents the passivity based control methods. Simulation results are presented in section IV, and section V concludes the paper.

## 2. THE DYNAMIC EQUATIONS OF THE ROBOT MANIPULATOR

The general equation of motion for an open chain manipulator can be expressed through the application of the lagrangian equation. The Denavit-Hartenberg matrix representation is used to describe the displacement between the neighbouring link coordinate frames to obtain the link kinematic information. The Lagrange equation (1) describes the manipulator motion.

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \tau_i \quad (1)$$

Where  $L=E-P$ .  $E$  and  $P$  are respectively the kinetic and potential energies of the manipulator.  $\tau_i$  represents the torque developed by the  $i$ -th joint.

The kinetic energy for each link is written as:

$$E_{ci} = 1/2 (mV_G^2 + \Omega(I.\Omega)) \quad (2)$$

Where:  $V_G$  is the linear velocity of the center of mass

$\Omega$  is the angular velocity

$I$  is the inertia tensor

$m$  is the mass of the link

The potential energy of each link is written as

$$G_i = m_i g h_i \quad (3)$$

$m_i$  : mass of the  $i$ -th link

$h_i$  : position of the center of mass expressed in the world coordinate

The use of Lagrange equation yield directly to the dynamic model of the robot manipulator

$$\tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + V(\dot{q}) \quad (4)$$

Beside the terms found, we have taken into consideration the friction term composed of two types of frictions namely:

$$V(\dot{q}) = K_{fs} \text{sgn}(\dot{q}) + K_v(\dot{q}) \quad (5)$$

The dynamic equations relate torque to position, velocity and acceleration. The solution of these equations allows the motions of the manipulator to be found. These dynamic equations are normally expressed in a matrix form in order to obtain the necessary information for control. The system considered in this study is a three-degree-of-freedom manipulator having three parallel axes of rotation suspended to the ceiling to the ceiling as it is depicted by Figure 1.

The following properties make the high nonlinear equations (4) a particular class of nonlinear systems, facilitating their study and design.

- 1) Boundedness and positivity of the inertia matrix  $D(q)$ :  $\bar{\delta}I \geq D(q) = D^T(q) \geq \underline{\delta}I, \forall q \in \mathbb{R}^n$  and some  $\bar{\delta} \geq \underline{\delta} > 0$ .
- 2) Independent control input for each degree of freedom.
- 3) Linear parametrization (4) can be written as

$$\tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + V(\dot{q}) = Y(q, \dot{q}, \ddot{q})\theta \quad (6)$$

where  $\theta$  is an  $r$ -dimensional vector containing the parameters of interest, and  $Y(q, \dot{q}, \ddot{q})$  is an  $n \times r$  matrix called regressor, containing known functions.

- 4) Skew symmetric matrix:  $q^T [\dot{D}(q) - 2C(q, \dot{q})]q = 0, \forall q \in \mathbb{R}^n$ .

## 3. THE PASSIVITY BASED ADAPTIVE CONTROL

Passivity based adaptive controllers have several advantages over those based on the linearization. These controllers avoid the inversion of the estimate of the inertia matrix and do not need measurement of the acceleration.

The dynamic equation  $\tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + V(\dot{q})$  define a passive mapping  $\tau \rightarrow \dot{q}$ ; i.e

$$\langle q | \tau \rangle = \int_0^T \dot{q}^T \tau dt \geq -\beta \quad (7)$$

for some  $\beta > 0$ , for all  $T$ .

Now, we investigate the control scheme that exploit the skew symmetry (property 4). This control does not lead to a linear system even if all the parameters are known.

Consider now the equation

$$\tau = \hat{M}(q)u + \hat{C}(q, \dot{q})\dot{q}_r + \hat{g}(q) + \hat{V}(\dot{q}) - F_p e - F_v s \quad (8)$$

using property (3), we can write

$$\hat{M}(q)u + \hat{C}(q, \dot{q})\dot{q}_r + \hat{g}(q) + \hat{V}(\dot{q}) = Y(q, \dot{q}, \dot{q}_r, u)\hat{\theta} \quad (9)$$

such that

$\dot{q}_r = \dot{q}^d - \Lambda e$ ,  $\dot{q}_r$  is the reference velocity,  $\dot{q}^d$  the desired velocity and  $\Lambda$  is a positive diagonal matrix.

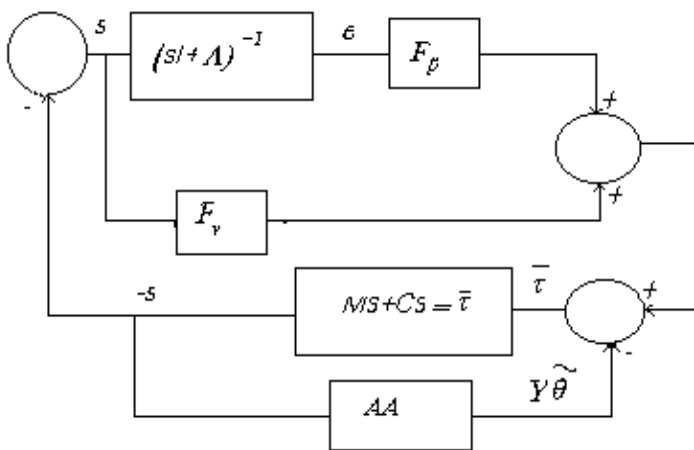


Fig. 1. The closed loop system

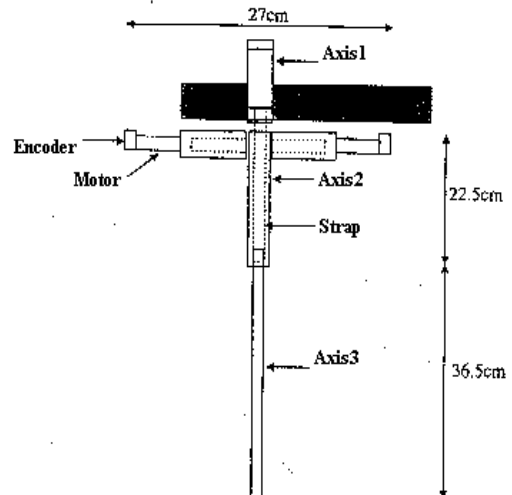


Fig. 2. The 3-dof robot manipulator

$$u(t) = \ddot{q}^d - \Lambda \dot{e} = \ddot{q}_r \tag{10}$$

$$s(t) = \dot{q}(t) - \dot{q}_r(t) = \dot{e}(t) + \Lambda e(t) \tag{11}$$

This last equation may be put in the form

$$e = (sI + \Lambda)^{-1} s \tag{12}$$

$\Lambda$  is chosen such that  $(sI + \Lambda)^{-1}$  is strictly proper and stable. The closed loop may be expressed as (see Fig. 1) and gives the following equation

$$\tau = Y(q, \dot{q}, \ddot{q}_r, u)\hat{\theta} - F_p e - F_v s \tag{13}$$

Thus

$$F_p e + F_v s = -\tau + Y(q, \dot{q}, \ddot{q}_r, u)\hat{\theta} + Y(q, \dot{q}, \ddot{q}_r, u)\tilde{\theta} \tag{14}$$

where  $\tilde{\theta} = \hat{\theta} - \theta$

taking into account equation (4) and replacing in equation (14),

$$F_p e + F_v s = \bar{\tau} + Y(q, \dot{q}, \ddot{q}_r, u)\tilde{\theta} \tag{15}$$

where

$$\bar{\tau} = -[M(q)\dot{s} + C(q, \dot{q})s] \tag{16}$$

To show the passivity of the system we need simply to show that the three blocs composing the closed loop system are passive. Making use of equation (7), we can easily prove the passivity of each of the three blocs. We conclude consequently that  $s \in L_2^n \Rightarrow e \in L_2^n \cap L_\infty^n, \dot{e} \in L_2^n, e$  is continuous and  $e \rightarrow 0$

when  $t \rightarrow \infty$ . Hence, the closed loop system is asymptotically stable

#### 4. SIMULATION RESULTS

In order to observe the performance of the passivity control, we have done simulations Matlab5/SIMULINK, on a three-degree-of-freedom robot manipulator, which consists of three links, coupled through a gear train to dc motors (Fig. 2).

The values used in the simulations were  $l_1 = 27cm; l_2 = 22.5cm, l_3 = 36.5cm, m_1 = 0.78, m_2 = 0.18, m_3 = 0.07$

The robot axes are considered to be cylindrical having the following inertial matryices

$$I_{G1} = \begin{bmatrix} B_1 & & \\ & A_1 & \\ & & B_1 \end{bmatrix}, I_{G2} = \begin{bmatrix} A_2 & & \\ & B_2 & \\ & & B_2 \end{bmatrix},$$

$$I_{G3} = \begin{bmatrix} A_3 & & \\ & B_3 & \\ & & B_3 \end{bmatrix}$$

with  $A_i = m_i \frac{R_i^2}{2}$  and  $B = \frac{m_i}{12}(3R_i^2 + h_i^2)$   $m_i, R_i, h_i$  represent the mass, the radius and the hight of the link respectively.

The results given in Figures (3) and (4), show how best is the control based on passivity property. when compared to the linearization control method, this former is more robust in the sense that the latter control law seeks to obtain a decoupled linearized system. However, the control law based on passivity does not seek to linearize the system but to assure the passivity of the system of its closed loop form. The dynamic reponse of

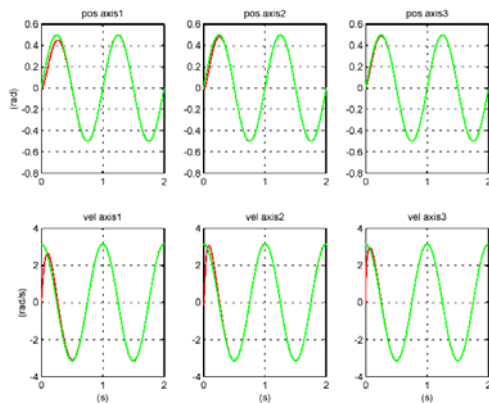


Fig. 3. Position and velocity curves

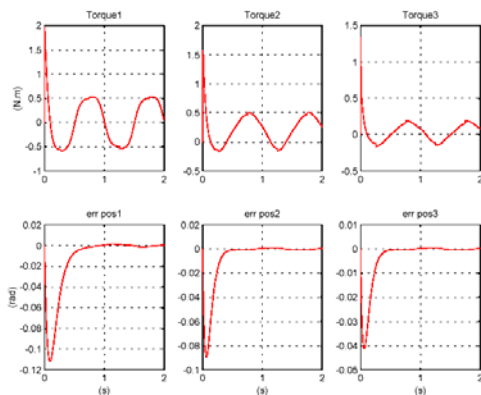


Fig. 4. Torque and position errors curves

the system and consequently the performances obtained by this approach are well suited and appreciated. Figure (4) shows the curves for the torques developed and the position errors.

## 5. CONCLUSION

The passivity based control algorithm applied to a three degree-of-freedom robot manipulator has been presented in this note. We have investigated the passivity principle and its application to a certain class of robot manipulators. From the curves we conclude that this approach is well suited for systems where a smoother behavior of the tracking error and parameter estimates are desirable.

## REFERENCES

- [1] Brogliano, B., Landau, I., and Lozano, R., "Adaptive motion control of robot manipulator: A unified approach based on passivity", *Int. J. Robust Nonlinear Contr.*, 1991, vol. 1, 187-202
- [2] Lozano, R. and Canudas, C., "Passivity based adaptive control for mechanical manipulators using LS-type estimation", *IEEE Trans. Automat. Contr.*, 1990, vol. 35, 1363-1365
- [3] Ortega, R. and Spong, M., "Adaptive motion control of rigid robots: A tutorial", *Automatica*, 1989, vol. 25, 877-888

- [4] Slotine, J.J., and Li, W., "Applied nonlinear control", Englewood Cliffs, NJ: Prentice-Hall, 1990
- [5] Shiriaev, A., Pogromski, A., Ludvigsen, H. and Egeland, O., "On global properties of passivity-based control of an inverted pendulum", *Int. J. of Robust and Nonlinear Control*, 2000, 10: 283-300.
- [6] P. Li and R. Horowitz, "Passive velocity field control of hierarchical manipulators", *IEEE Trans. on Robotics and Automation*, Vol. 15, No. 4, pp. 751-763 (1999).