MARQUARDT ALGORITHM ADAPTIVE FILTERING FOR NOISE CANCELLING

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Abstract

This paper presents a technique for cancelling undesired noise in communication systems. An original use of Marquardt algorithm in adaptive filtering field is presented. The advantage of the proposed algorithm is to do without well adjusting the step-size parameter in advance as for the famous LMS algorithm. Simulation and experimental results shows the effectiveness and robustness of the proposed algorithm in adaptive filtering field.

KEYWORDS: Adaptive filter, LMS algorithm, Marquardt algorithm, Noise Cancellation System.

I. Introduction

Adaptive filtering is applied in diverse fields like radar, sonar, seismology, biomedical engineering and communication [1]. Although these various applications are very different in nature, one common feature can be noted: an input vector and a desired response are used to compute an estimation error which is used, in turn, to control the values of a set of adjustable filter coefficients.

This paper deals with noise cancellation in communication systems. The adaptive filter is used to cancel undesired noise contained in a primary signal. The latter contains the desired signal s(n) and the undesired noise v(n) as illustrated in Fig. 1.

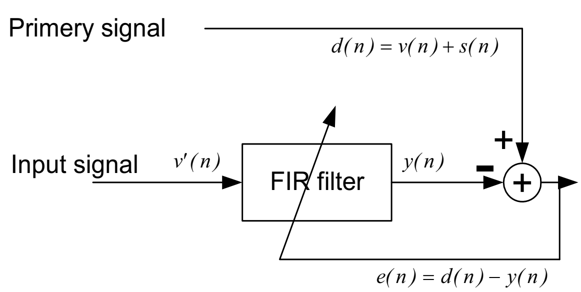

Figure 1: Adaptive filter noise cancellation

The input signal v'(n) is a correlated noise with v(n). v'(n) and s(n) are uncorrelated. Then the adaptive filter adjusts the input noise v'(n) to produce y(n) that approximates the noise v(n). Therefore, the noise component v(n) in the primary signal is canceled by y(n). The error signal e(n), gradually, converges to an approximation of the desired signal s(n) by using an adequate optimization method.

The well-known Least-Mean-Square (LMS) algorithm [2] is applied to many applications in communication systems. It is an important member of the family of stochastic gradient- based algorithm.
Several applications of the LMS algorithm presented and dealing with adaptive noise cancelling were used with finite impulse response (FIR) filters. A significant feature of the LMS algorithm lies in its simplicity and robustness in many applications. But care must be taken of the step-size parameter μ because the latter controls the convergence rate of the FIR filter coefficients and the convergence time is inversely proportional to μ. Generally, the step-size μ used in the LMS algorithm is fixed at a small value and must be defined in advance.
To accelerate the convergence rate of the LMS algorithm, a variable step size is needed [3]. Marquardt algorithm which is widely used in the identification field [4] is proposed. It is the association of the stochastic gradient method and the Newton method. It helps reach an optimal solution with variable step-size without the need to well adjust the value in advance.

This paper is organized as follows: in section II the principle of noise cancelling by FIR adaptive filtering is proposed. Section III deals with the proposed Marquardt algorithm. Comparative results between the proposed algorithm and the LMS technique are given in section IV. They show the effectiveness of the proposed Marquardt algorithm. Section V concludes the paper.

II. Principle of adaptive filter noise canceling

From the Fig. 1, we can write,

$$e(n)=d(n)-y(n)=s(n)+v(n)-y(n) \qquad (1)$$

Thus, the mean square error is given by,

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$$E\left[e(n)^2\right] = E\left[\left(s(n)+v(n)-y(n)\right)^2\right]$$
$$= E\left[s(n)^2\right] + E\left[\left(v(n)-y(n)\right)^2\right] + \qquad (2)$$
$$+ 2E\left[s(n).\left(v(n)-y(n)\right)\right]$$

The signal $s(n)$ is not correlated neither with $v(n)$ nor with $y(n)$. So,

$$E\left[s(n).\left(v(n)-y(n)\right)\right] = 0 \qquad (3)$$

Then

$$E\left[e(n)^2\right] = E\left[s(n)^2\right] + E\left[\left(v(n)-y(n)\right)^2\right] \qquad (4)$$

The minimum mean square error becomes,

$$\min E\left[e(n)^2\right] = E\left[s(n)^2\right] + \min E\left[\left(v(n)-y(n)\right)^2\right] \qquad (5)$$

Or,

$$e(n) - s(n) = v(n) - y(n)$$

$$\Rightarrow \min E\left[\left(e(n)-s(n)\right)^2\right] = \min E\left[\left(v(n)-y(n)\right)^2\right]$$

So, minimizing the mean output error which means that when the output filter converges to the undesired noise $v(n)$, the output error $e(n)$ tends to the clean signal $s(n)$.

III. Marquardt technique

The output of the transversal p order FIR filter structure is ,

$$y(n) = \underline{w}^T\,\underline{v}'(n) \qquad (6)$$

where $\underline{w}^T = [w_0 \ \ w_1 \ \ ....w_{p-1}]$ is the parameter vector of the filter and $\underline{v}'(n)^T = [v'(n) \ \ v'(n-1) ....v'(n-p+1)]$ is the noise input signal. The sample index is n. The data set is composed of N data pairs {d(n),v'(n)}.

Defining the output error:

e (n)= d(n)-y(n)

The optimal solution of parameters filter is obtained by minimization of the quadratic criterion given by (7),

$$J = \sum_{n=p}^{N} e(n)^2 \qquad (7)$$

The different coefficients of the filter are updated at the i[th] iteration by (8):

$$\underline{w}_{i+1} = \underline{w}_i - \left\{\left[J''_{\underline{w}_i\underline{w}_i} + \lambda I\right]^{-1} J'_{\underline{w}_i}\right\} \qquad (8)$$

Where $J'_{\underline{w}_i}$ and $J''_{\underline{w}_i\underline{w}_i}$ are the gradient and the Hessian, respectively. They are given by,

$$J'_{\underline{w}_i} = -2\sum_{n=p}^{N} e(n)\sigma_{\underline{w}_i}(n) \qquad (9)$$

$$J''_{\underline{w}_i\underline{w}_i} \approx 2\sum_{n=1}^{N} \sigma_{w_i}(n)\sigma_{w_i}(n)^T \qquad (10)$$

The sensitivity function $\sigma_{\underline{w}_i}$ with respect to the different coefficients vector of the filter parameters is given by (11),

$$\sigma_{\underline{w}_i}(n) = \frac{\partial \underline{y}(n)}{\partial \underline{w}_i} \qquad (11)$$

λ is the monitoring parameter which can be fixed at any value at the beginning of the algorithm. λ can vary in two ways;

1. When the distance between the proposed output filter and the desired signal is important. In this case, no optimal solution is any near. Therefore, the value of λ must be increased. So we obtain the stochastic-gradient behavior.
2. When the output filter converges toward the undesired noise signal bringing the optimal solution closer. The value of λ must be decreased and the Newton algorithm behavior is obtained.

The Marquardt algorithm is described below:

Initialization: (first iteration)
- Parameter vector of the filter w
- Monitoring parameter λ
- Computing of: $\underline{y}$, $\underline{e}$, J , $J'_{\underline{w}_i}$ and $J''_{\underline{w}_i\underline{w}_i}$
- Defining termination conditions

While termination conditions not met do
- Updating the filter parameter vector w
- Computing: $\underline{y}$, $\underline{e}$, J , $J'_{\underline{w}_i}$ and $J''_{\underline{w}_i\underline{w}_i}$
- Convergence test and adjustment of the monitoring parameter λ for each iteration.

End while

IV. Noise cancelling results

IV.1. Simulation results

A 32 order adaptive FIR filter is used. A white noise and colored noise are considered and added to the clean signal s(n), which is in the first case a sine wave at 8 KHz frequency and in the second case is a speech signal (.wav). When applying the LMS algorithm, the step-size parameter used is equal to 0.01. But when applying Marquardt algorithm, the monitoring parameter is initialized at any value and then a self adjusting will occur providing more robustness of the algorithm.

The obtained results are shown in figures 2, 3, 5 and 6. Figure 2 presents the noisy signal with additive white noise, the convergence rate of the squared criterion of Marquardt algorithm, comparison evolution of the error curve which is in this case the sine wave and obtained by using Marquardt and LMS algorithms.

The self adjusting of the monitoring parameter λ ensures a faster convergence rate which is highly demanded in real applications, such as noise cancelling in real time speech processing.

The rejection of the additive white noise from the primary signal d(n) is efficient when using the Marquardt algorithm. So, the output of the adaptive filter presents a good estimation of the additive white noise used.

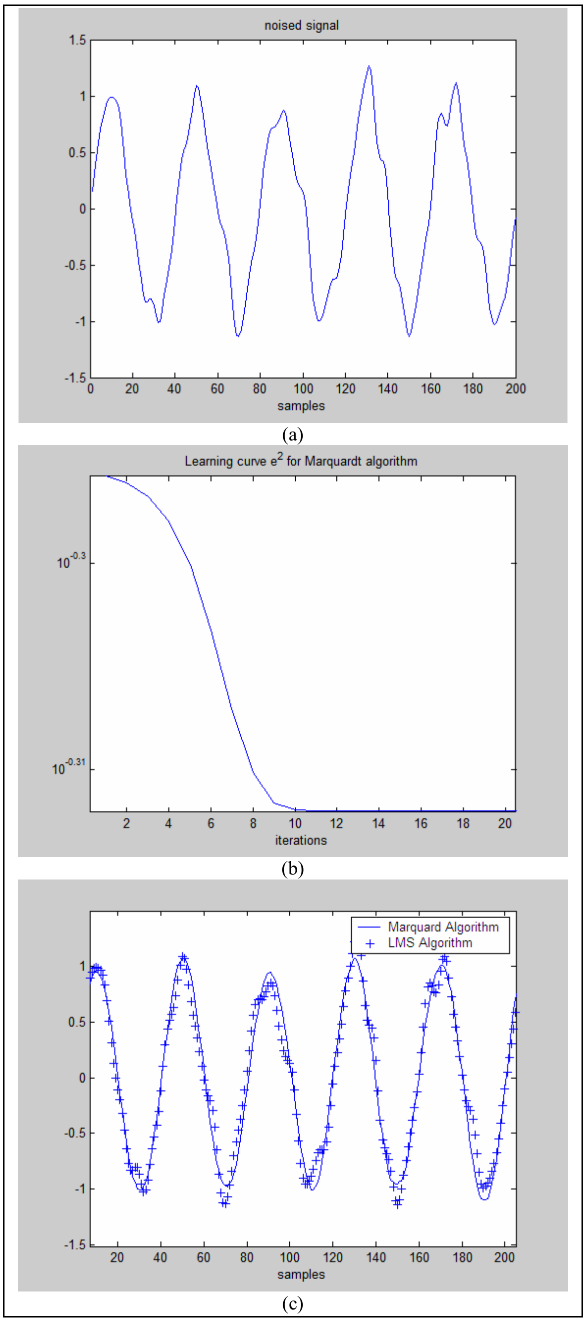


Figure 2. White noise canceling adaptive filter. S(n) with additive white noise (a); Convergence rate of squared criterion (Marquardt) (b); The obtained signal with Marquardt and LMS algorithms (c).

Figure 3 presents the obtained results in the case of an additive colored noise to the signal s(n).

In the case of an additive colored noise, the error signal which must converges to the clean signal after applying Marquardt algorithm or LMS algorithm, is presented. The error signal obtained with the proposed algorithm is closer to the clean signal then the one obtained by the LMS algorithm. To get better results with LMS technique, the step-size parameter μ have to be well adjusted.

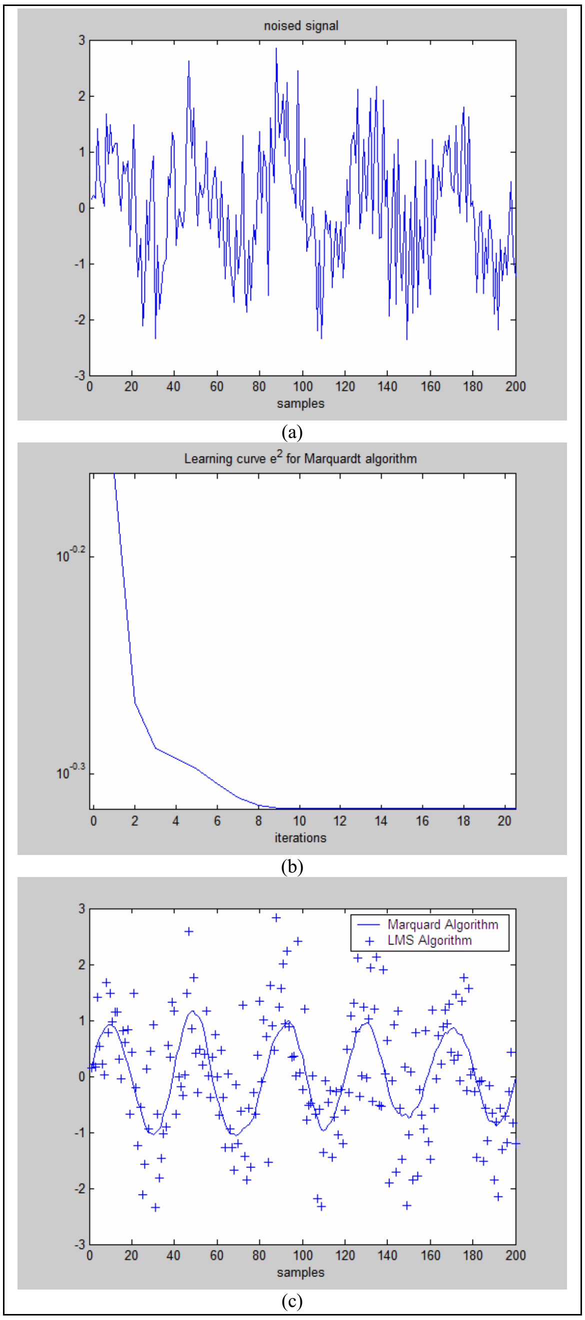


Figure 3: Colored noise cancellation. S(n) with additive colored noise (a); Convergence rate of squared error (Marquardt) (b); Error signal (estimated signal) with Marquardt and LMS algorithms (c)

IV.2 Experimental Results

The results obtained from the application of the Marquardt and LMS algorithms using speech signal as adaptive filter input signal are presented. The speech signal (.wav) is from TIMIT database and presented in figure 4. White and colored noises are added to the input signal. Figure 5 presents, respectively, the speech signal with additive white noise, the estimated speech signal (error signal) with the Marquardt algorithm and the estimated speech signal with the LMS algorithm.

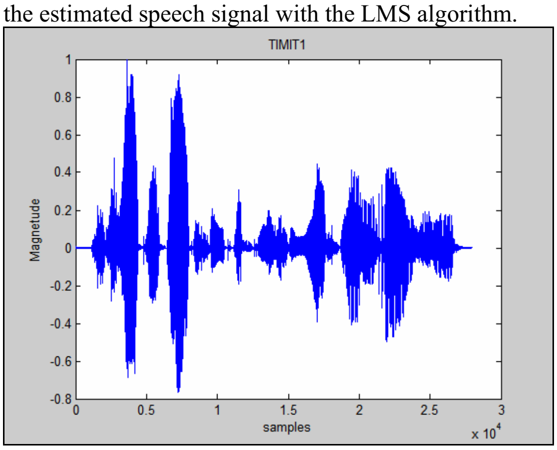
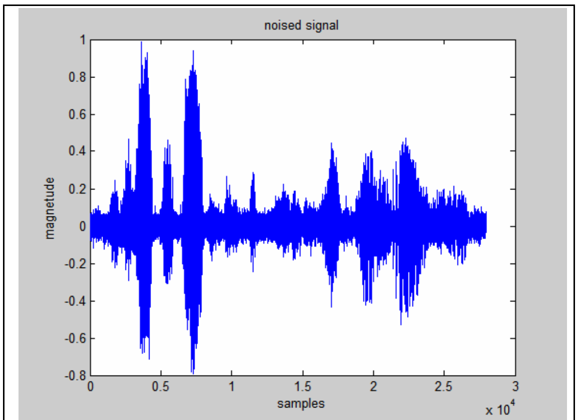
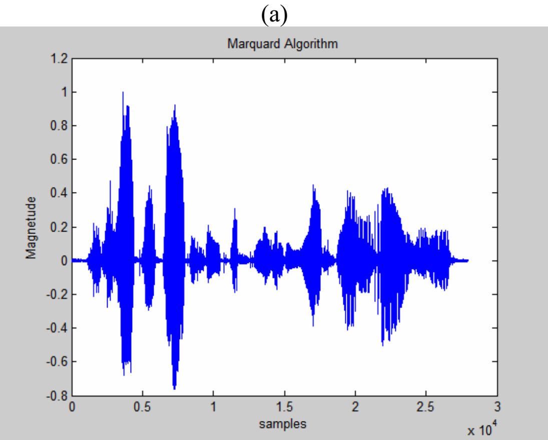


Figure 4 clean speech signal

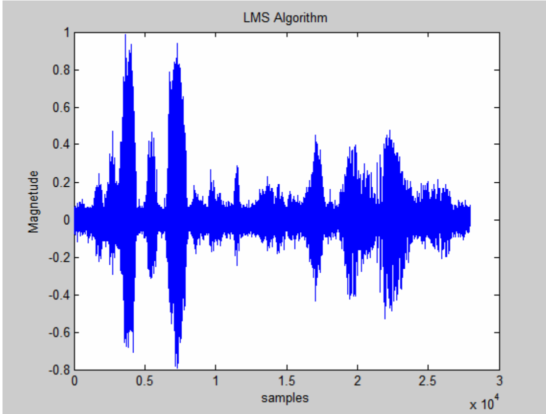
Better results could be obtained with the LMS algorithm if a well adjusting of the step-size μ is done but with the Marquardt algorithm no care is needed when defining the monitoring parameter λ.



(a)



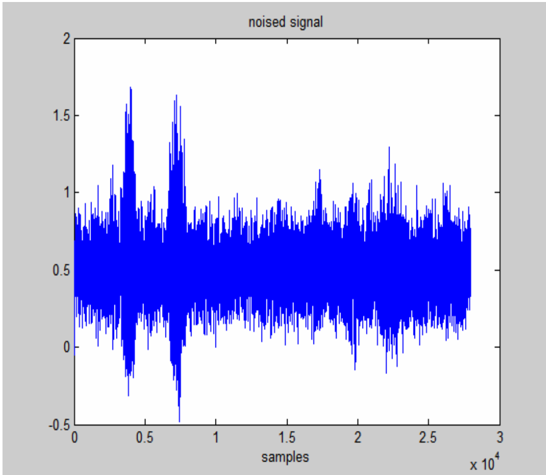
(b)



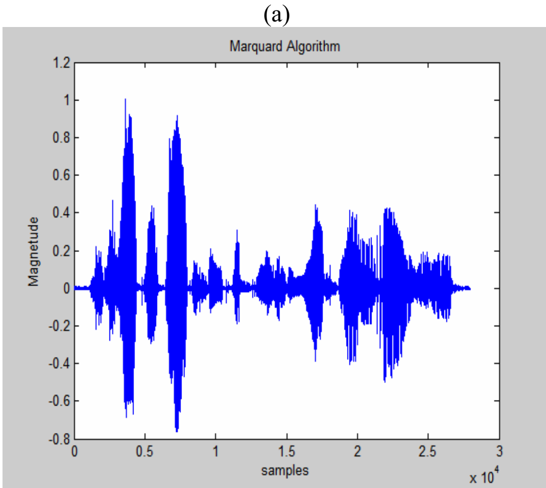
(c)

Figure 5: Marquardt (b) and LMS (c) results comparison with additive white noise to the speech signal (a).

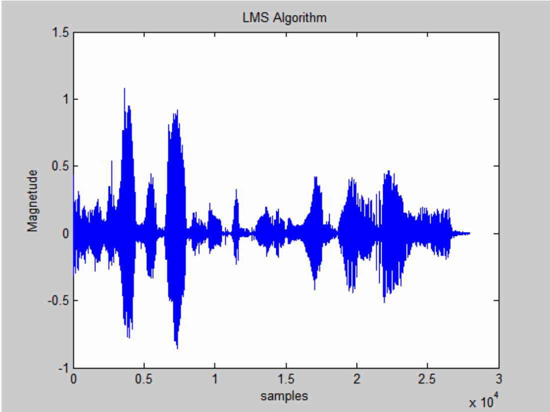
Figure 6 presents, respectively, the speech signal with additive colored noise, the estimated speech signal (error signal) with the Marquardt algorithm and the estimated clean speech signal with the LMS algorithm.



(a)



(b)



(c)

Figure 6: Marquardt (b) and LMS (c) results comparison with additive colored noise to speech signal (a).

IV. Conclusion

The well-known LMS algorithm is widely used in adaptive filtering field. The main feature of the LMS algorithm lies in its simplicity and robustness in many applications. But care must be taken when defining the step-size parameter μ because the latter controls the convergence rate of the FIR filter coefficients and the convergence time is inversely proportional to μ.

In this paper an original use of Marquardt technique is proposed. The Marquardt algorithm is a member of the stochastic gradient methods as LMS method. It permits to overcome the need of well adjusting the step-size parameter of LMS algorithm and a self adjusting monitoring parameter λ is used instead. Marquardt algorithm outperforms LMS algorithm in terms of convergence rate and complexity. These performances and effectiveness show its ability to solve many communication problems.

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