

## Frequency Domain Characterization of Intermodulation Distortion in Quasi-Memoryless Microwave Amplifiers

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**Abstract** — New formulas for estimating the spectral regrowth at the output of RF Power Amplifiers (RFPAs) have been proposed. The formulas predict the average output power spectral density (PSD) of fifth-order quasi-memoryless RFPAs, excited with uncorrelated tones. The efficiency of the proposed approach is demonstrated by predicting the spectral regrowth in a commercial RFPAs for different input power levels. The obtained results were compared to those of a harmonic balance simulation and have shown a good accuracy and time efficiency.

**Index terms** — Spectral regrowth, intermodulation distortion, power amplifiers, multitone, power series.

### I. INTRODUCTION

In today's technological drive towards ever higher performing communication systems, complex digitally modulated signals such as wideband CDMA and multicarrier signals are being used more and more. Consequently, the characterization of nonlinear RFPAs using conventional single and two-tone tests is no longer sufficient to predict the circuit's response in its final operation regime. For this reason, a number of uncorrelated tones that approximate digitally modulated wideband signals are used to excite RFPAs under test. However, the generation of such tones often requires devoted setups and suffers from several limitations such as the number of tones and the randomization of their phases. A reasonable alternative to the multitone test is the development of techniques to predict the spectral regrowth by means of data collected from standard single and two-tone tests.

Time domain techniques such as transient and envelop simulations are often used to simulate nonlinear RFPAs. However, the results of this type of simulations suffer from accuracy problems related to the time window and resolution. A lot of work has been done such as in [1] - [3], to develop frequency domain approaches to predict the spectral regrowth. Although many of these approaches support several types of excitations such as CDMA, OFDM and AWGN signals and/or able to handle memory effects, all of them are restricted to third-order nonlinearity or necessitate certain convolution, Fourier Transformation and/or iterative calculations. In this paper we extend the technique proposed in [4], to the analysis of fifth-order quasi-memoryless RFPAs.

### II. FORMULATION

Consider a multitone excitation of  $n$  equally spaced tones with constant amplitude  $A$  and frequency step  $\Delta\omega$  such that

$$x(t) = \text{Re} \left( \sum_{i=1}^n A e^{j(\omega_i t + \phi_i)} \right) \quad (1)$$

where  $\omega_i = \omega_1 + (i-1)\Delta\omega$  for  $1 \leq i \leq n$ , and  $\omega_1$  is the frequency of the first input tone in the spectrum of  $x(t)$ . In order to approximate wideband digitally modulated signals to  $x(t)$  the phases of the input tones  $\phi_i$ ,  $i = 1, 2, \dots, n$ , are considered as independent random variables, equally and uniformly distributed over  $0$  to  $2\pi$ .

A common characterization of a quasi-memoryless RFPAs is based on its amplitude-amplitude modulation (AM-AM) and its amplitude-phase modulation (AM-PM) conversions. To predict the spectral regrowth of a fifth-order nonlinear RFPAs, the complex envelopes  $\tilde{x}(t)$  and  $\tilde{y}(t)$  of the input and output pass band signals are related with  $\tilde{y}(t) = \tilde{x}(t)G(r)$  where  $r = |\tilde{x}(t)|$  and  $G(r)$  is the complex gain of the RFPAs, with  $|G(r)|$  is its gain (AM-AM) and  $\angle G(r)$  is its output phase shift (AM-PM). For the analysis of the pass band distortion,  $G(r)$  can be represented by a complex power series with odd terms only. A least square curve fitting of the AM-AM and AM-PM conversions allows the determination of the complex-valued coefficients of the series,  $k_1$ ,  $k_3$ , and  $k_5$ . Hence, the RFPAs input-output relationship becomes

$$\tilde{y}(t) = k_1 \tilde{x}(t) + k_3 \tilde{x}(t) |\tilde{x}(t)|^2 + k_5 \tilde{x}(t) |\tilde{x}(t)|^4 \quad (2)$$

The spectrum of  $y(t)$  is composed of an amplified version of the input spectrum combined with several IM products located at different frequency points  $\omega_b$ . The resulting tone at a given frequency point in the pass band signal is a combination of the IM products generated by the linear, third and fifth-order nonlinear terms of the complex power series. To calculate the overall amplitude of the resulting tone, the phase relationship between the IM products must be established first. In fact, not all the IM products, falling in the same frequency point are necessarily in phase. Since the input phases are random and uniformly distributed over  $0$  to  $2\pi$ , only the expected power of the generated tone can be calculated [1]. To do so, it is necessary to classify the IM products according to their phase relationship and summing them linearly or

squared by consequence [4]. After identifying the third- and fifth-order groups of arrangements, an advanced combinatory analysis technique, called generating function [5], has been used to calculating the number of frequency arrangements in each group. This has led to the determination of the output PSD  $S_y(\omega_b)$  such that

$$S_y(\omega_b) = \frac{1}{2} [L(X, n, b) + R(X, n, b) + S(X, n, b)] \quad (3)$$

$$L(A, n, b) = |k_1 A U_1(n, b) + k_3 A^3 (2n-1) + k_5 A^5 (6n^2 - 9n + 4)|^2 \quad (4)$$

$$R(A, n, b) = |k_3 A^3 + k_5 A^5 (6n-7)|^2 T_2(n, b) + |2k_3 A^3 + k_5 A^5 6(2n-3)|^2 T_4(n, b) \quad (5)$$

$$S(X, n, b) = |k_5|^2 A^{10} \left\{ \begin{aligned} &F_{11}(n, b) + 9F_{12}(n, b) + 36F_{13}(n, b) \\ &+ 4F_{14}(n, b) + 36F_{15}(n, b) + 144F_{16}(n, b) \end{aligned} \right\} \quad (6)$$

where  $U(n, b) = 1$  if  $1 \leq b \leq n$  and 0 otherwise.  $T_2(n, b)$ ,  $T_4(n, b)$  and  $F_{11}(n, b)$  to  $F_{16}(n, b)$  are presented in [4] for  $3 - 2n \leq b \leq 3n - 2$ .

### III. RESULTS

To illustrate the efficiency of the proposed formulas, a commercial RFPA with a  $P_{1dBcomp} = 16$  dBm has been used. The coefficients  $k_1$ ,  $k_3$  and  $k_5$  of its complex gain are found to be  $k_1 = 14.9740 + j0.0519$ ;  $k_3 = -23.0954 + j4.968$  and  $k_5 = 21.3936 + j0.4305$ . Fig. 1 shows the AM-AM and AM-PM conversions of the RFPA, measured at  $f = 2$  GHz, with their corresponding fitted polynomials in dashed lines. Also the figure shows that the curve fitting is valid for a maximum total input power of  $\approx 24$  dBm.

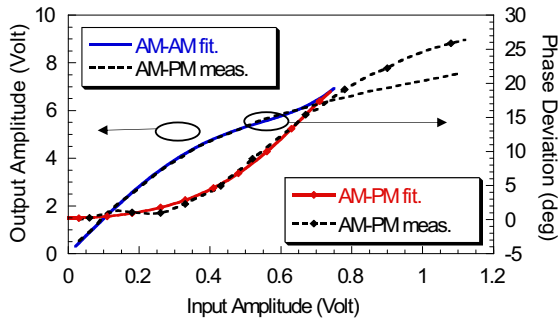


Fig. 1. AM-AM and AM-PM conversions.

The input signal is composed of 10 uncorrelated tones starting from  $f_i = 2$  GHz with  $\Delta_f = 1$  MHz. Fig. 2 and Fig. 3 show the predicted  $S_y(\omega_b)$  for an input power of 0 dBm per tone ( $P_{in,tot} = 10$  dBm  $\rightarrow$  6 dB back off) and 9 dBm per tone ( $P_{in,tot} = 19$  dBm  $\rightarrow$  3 dB above  $P_{1dBcomp}$ ) respectively. For comparison purpose, harmonic balance simulations have been performed using ADS with 20 different sets of random phases for each input power. The obtained results reveal a good agreement between the predicted output power and the averaged ADS simulation data.

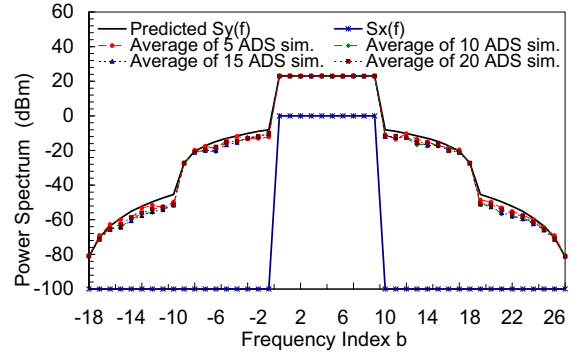


Fig. 2. RFPA in/out PSDs for  $n = 10$  and  $P_{in} = 0$  dBm.

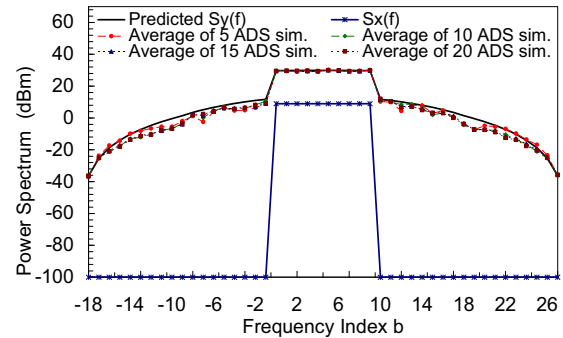


Fig. 3. RFPA in/out PSDs for  $n = 10$  and  $P_{in} = 9$  dBm.

### IV. CONCLUSION

The suggested formulas represent a useful approach to predict the distortion in fifth-order quasi-memoryless RFPAs, excited by  $n$  uncorrelated equally spaced tones. The obtained results revealed the robustness and time efficiency of the developed expressions, compared to time domain or harmonic balance simulations. Furthermore, this approach can be used with Volterra series to predict the spectral regrowth in the RF nonlinear amplifiers with memory.

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