

ROBUST NONLINEAR SPEED CONTROL OF PM SYNCHRONOUS MOTOR

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Abstract: This paper deals with robust nonlinear speed control of permanent magnet synchronous machine. The controller is based on a one-step-ahead continuous time minimization of the predicted tracking errors and no online optimization is required. An integral action is incorporated in the loop to increase the robustness of this control scheme. The proposed nonlinear controller permits to achieve asymptotic speed tracking in presence of the unknown load torque and resistance variations. Simulations are performed to validate the tracking performance of the proposed controller.

Keywords: Nonlinear predictive control, PMSM, integral action and robustness.

1. Introduction

Permanent magnet synchronous motors (PMSM) are being increasingly used in a wide range of applications due to their high power density, large torque to inertia ratio, and high efficiency. The PMSM is very similar to the standard wound rotor synchronous machine except that the PMSM has no damper winding and excitation is provided by a permanent magnet instead of a field winding. Hence, a $d-q$ model of PMSM can be derived from the well-known model [1, 2] of the synchronous machine with the equations of the damper windings and field current dynamics removed. However, the dynamic model of PMSM is highly nonlinear, because of the coupling between the motor speed and electrical quantities such as the $d-q$ axis currents.

The traditional field orientation [1] has been widely adopted to linearize the nonlinear model and has obtained significant achievement. However, for a higher performance requirement, such as robots and machine tools, this method may be not sufficient during the speed transient.

In recent years, various nonlinear control methodologies have been applied to PMSM in order to design a controller directly by considering the nonlinear PMSM dynamics. Sliding mode variable structure has been studied by many researchers due to its favorable advantages, such as its insensitivity to parameter uncertainties and external disturbances [3]. Only the bounds of the uncertainties are needed. The robustness of this control scheme is guaranteed, but the worst drawback is the chattering phenomenon, which limits the application of this approach. Although, a robust speed

control of PMSM using boundary layer integral sliding mode control is presented in [4] to reduce the chattering phenomenon.

State feedback linearization and input-output feedback linearization [5, 6] have been thoroughly studied over the last 20 years, by which means the original nonlinear model is transformed to linear model through a proper coordinate transformation. Thus, almost all the well developed linear control techniques might be applied. This method has proved to be effective to control PMSM [7]. However, as it is based on the exact cancellation of nonlinear elements of the system model, when there are parameter uncertainties or unknown disturbances, the desired results may not be obtained. For this reason some researchers have added adaptive mechanism to estimate unknown or variable parameters [8].

In [9], the combination of the exact input-output linearization and back stepping approach leads to a nonlinear adaptive speed controller of PMSM. Here also, the adaptive mechanism is used to overcome the sensitivity of the controller to parameter uncertainties and load torque disturbances.

An energy-based controller design technique for Hamiltonian system has been applied to the speed regulation of PMSM [10]. A globally convergent closed-loop system is obtained with a linear observer that estimates the load torque. The robustness of this control scheme with respect to parameter uncertainties (especially resistance variation) has not been reported.

In this paper, a one-step ahead predictive control scheme is developed for speed tracking control of PMSM. The asymptotic stability of the resulting closed-loop

system is guaranteed and good robustness with respect to resistance variation is obtained. It is known that in the literature, the load torque induces a steady state error in the speed tracking dynamic. To eliminate this steady state error an integral action is introduced in the loop. Thus, the load torque estimator is not needed in this control scheme.

The organization of this paper proceeds as follows. First, a mathematical model of PMSM is presented. Then a one-step-ahead predictive controller is developed to allow speed tracking of the desired reference trajectory. Next, the integral action is introduced in the loop to enhance the robustness of the control scheme with respect to unknown time-varying load torque. Finally, the simulation results are given, looking both at the nominal and mismatched model.

2. Modeling of PMSM

The machine considered is a surface mounted PMSM and the nonlinear equation in synchronous d - q reference frame can be represented as follows [1, 2, 11, 13]:

$$\frac{dx}{dt} = f(x) + Gu \quad (1)$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} i_d \\ i_q \\ \Omega \end{bmatrix}; u = \begin{bmatrix} u_d \\ u_q \end{bmatrix}; G = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0 \end{bmatrix}$$

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} = \begin{bmatrix} -\frac{R}{L_d} x_1 + p \frac{L_q}{L_d} x_2 x_3 \\ -\frac{R}{L_q} x_2 - p \frac{\Phi}{L_q} x_3 - p \frac{L_d}{L_q} x_1 x_3 \\ -\frac{B}{J} x_3 + \frac{3}{2} p \frac{\Phi}{J} x_2 - \frac{T_L}{J} \end{bmatrix}.$$

The parameters used in these equations are defined as follows:

- u_d, u_q : Stator voltages in direct and quadrature axes,
- i_d, L_d : Current and inductance in the direct axes;
- i_q, L_q : Current and inductance in the quadrature axes;
- R : Stator resistance;
- Ω : Mechanical speed of motor;
- p : Number of pole pairs;
- Φ : Flux created by the motor magnets;
- J : Moment of inertia;
- B : Viscous friction coefficient;
- T_L : Load torque;
- f_1, f_2, f_3 : Nonlinear terms in PMSM model.

The output of this system is:

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} i_d \\ \Omega \end{bmatrix} = Cx \quad (2)$$

$$\text{where } C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

3. One step ahead predictive control

In this section, we employ the one-step-ahead predictive scheme [12] to design the controller for PMSM system by assuming all states are measurable. In order to find the current control $u(t)$ that improves the tracking error along a fixed interval, the output tracking error: $e(t) = y(t) - y_{ref}(t)$ is used and the predicted cost function to be minimized is given by:

$$J = \frac{1}{2} \|y(t+h) - y_{ref}(t+h)\|_Q^2 + \frac{1}{2} \|u\|_R^2 \quad (3)$$

where $Q \in \mathfrak{R}^{2 \times 2}$ is a positive definite matrix, $R \in \mathfrak{R}^{2 \times 2}$ is a semi-positive definite matrix and $h > 0$ is a prediction horizon.

The one step predictor of the output can be written by using Taylor approximation. Indeed, the predicted d -axis current ($y_1 = i_d$) is:

$$y_1(t+h) = y_1(t) + h \dot{y}_1(t) = y_1(t) + h L_f y_1 + h L_g L_f y_1 u \quad (4)$$

where $L_f y_1$ and $L_g L_f y_1$ are defined in the appendix.

For the output y_2 (motor speed), the predicted signal is given by:

$$\begin{aligned} y_2(t+h) &= y_2(t) + h \dot{y}_2(t) + \frac{h^2}{2} \ddot{y}_2(t) \\ &= y_2(t) + h L_f y_2 + \frac{h^2}{2} L_f^2 y_2 + \frac{h^2}{2} L_g L_f y_2 u \end{aligned} \quad (5)$$

The predicted output can be written as:

$$y(t+h) = \begin{bmatrix} y_1(t+h) \\ y_2(t+h) \end{bmatrix} = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} h L_f y_1 \\ h L_f^2 y_2 + \frac{h^2}{2} L_g L_f y_2 \end{bmatrix} + \begin{bmatrix} h L_g y_1 \\ \frac{h^2}{2} L_g L_f y_2 \end{bmatrix} u$$

The reference signal should be also expanded to the same order by using Taylor approximation:

$$y_{ref}(t+h) = \begin{bmatrix} y_{ref1}(t+h) \\ y_{ref2}(t+h) \end{bmatrix} = \begin{bmatrix} y_{ref1}(t) \\ y_{ref2}(t) \end{bmatrix} + \begin{bmatrix} h \dot{y}_{ref1} \\ h \dot{y}_{ref2} + \frac{h^2}{2} \ddot{y}_{ref2} \end{bmatrix}$$

The predicted tracking error can be expressed as:

$$\begin{aligned} e(t+h) &= y(t+h) - y_{ref}(t+h) \\ &= e(t) + Z(t, h) - d(t, h) + \Lambda(h)u \end{aligned} \quad (6)$$

where

$$Z(t, h) - d(t, h) = \begin{bmatrix} h(L_f y_1 - \dot{y}_{ref1}) \\ h(L_f y_2 - \dot{y}_{ref2}) + \frac{h^2}{2}(L_f^2 y_2 - \ddot{y}_{ref2}) \end{bmatrix};$$

$$\Lambda(h) = \begin{bmatrix} \frac{h}{L_d} & 0 \\ 0 & \frac{3}{4} p \frac{\Phi h^2}{J L_d} \end{bmatrix}$$

The control signal that minimizes the cost function (3) is given (by setting $\frac{\partial J}{\partial u} = 0$):

$$u = -(\Lambda(h)^T Q \Lambda(h) + R)^{-1} \Lambda^T(h) Q (e(t) + Z(t, h) - d(t, h)) \quad (7)$$

a. Tracking performance:

Let $R=0$ in the equation (7), the control signal becomes:

$$u = -\Lambda^{-1}(h) Q (e(t) + Z(t, h) - d(t, h)) \quad (8)$$

and by using the equations:
$$\begin{cases} \dot{y}_1 = L_f y_1 + L_g y_1 u \\ \ddot{y}_2 = L_f^2 y_2 + L_g L_f y_2 u \end{cases}$$

with the above control signal, we get the tracking errors dynamics:

$$\begin{cases} \dot{e}_1 + \frac{1}{h} e_1 = 0 \\ \ddot{e}_2 + \frac{2}{h} \dot{e}_2 + \frac{2}{h^2} e_2 = 0 \end{cases} \quad (9)$$

where
$$\begin{cases} e_1(t) = y_1(t) - y_{ref1}(t) \\ e_2(t) = y_2(t) - y_{ref2}(t) \end{cases}$$

The above dynamics equations are linear and time invariant. Therefore, the tracking controller design technique without penalty on control leads to feedback linearization and we easily verify the asymptotic stability of the tracking errors dynamics of the overall system for any $h > 0$.

Note that, when the cost function is with penalty on control signal ($R \neq 0$) or in the presence of load torque a steady state error will be induced.

b. Integral action

An integral action is extensively used in servo control to eliminate steady state errors. Hence, in this work to eliminate the steady state error induced by model uncertainties or load torque disturbances, the integral action is incorporated in the loop. The controller with the integral action enhances the robustness of the control scheme with respect to model uncertainties and load torque disturbances.

Thus, we have to minimize the predicted position tracking error instead of speed tracking error. By defining the motor position variable $\theta(t)$, where $\dot{\theta}(t) = \Omega(t)$, the one-step-ahead predictor of the motor position is:

$$\begin{aligned} \theta(t+h) = & \theta(t) + h y_2(t) + \frac{h^2}{2} L_f y_2(t) \\ & + \frac{h^3}{6} L_f^2 y_2 + \frac{h^3}{6} L_g L_f y_2 u \end{aligned}$$

and the predicted reference signal is:

$$\theta_{ref}(t+h) = \theta_{ref}(t) + h \Omega_{ref}(t) + \frac{h^2}{2} \dot{\Omega}_{ref}(t) + \frac{h^3}{6} \ddot{\Omega}_{ref}(t)$$

Therefore, the predicted tracking error becomes:

$$\begin{aligned} e(t+h) &= \begin{bmatrix} y_1(t+h) - y_{ref}(t+h) \\ \theta(t+h) - \theta_{ref}(t+h) \end{bmatrix} = \begin{bmatrix} e_1(t+h) \\ e_\theta(t+h) \end{bmatrix} \quad (10) \\ &= e(t) + \bar{Z}(h, t) - \bar{d}(h, t) + \bar{\Lambda}(h) u \end{aligned}$$

where $e(t) = \begin{bmatrix} e_1(t) \\ e_\theta(t) \end{bmatrix}$; $\bar{\Lambda}(h) = \begin{bmatrix} h L_g y_1 \\ \frac{h^3}{6} L_g L_f y_2 \end{bmatrix}$ and

$$\begin{aligned} \bar{Z}(h, t) - \bar{d}(h, t) &= \\ & \begin{bmatrix} h(L_f y_1 - \dot{y}_{ref1}) \\ h e_2 + \frac{h^2}{2}(L_f y_2 - \dot{y}_{ref2}) + \frac{h^3}{6}(L_f^2 y_2 - \ddot{y}_{ref2}) \end{bmatrix} \end{aligned}$$

The cost function to minimize is:

$$\bar{J} = \frac{1}{2} \|e_\theta(t+h)\|_{\bar{Q}}^2 + \frac{1}{2} \|u\|_R^2 \quad (11)$$

By using the augmented predictive tracking error (10) and by setting $\frac{\partial \bar{J}}{\partial u} = 0$, the optimal solution is given by:

$$u = -(\bar{\Lambda}^T(h) \bar{Q} \bar{\Lambda}(h) + R)^{-1} \bar{\Lambda}(h)^T \bar{Q} (e_\theta(t) + \bar{Z}(h, t) - \bar{d}(h, t)) \quad (12)$$

c. Tracking performance

Let $R = 0$, the equation (12) becomes:

$$u = -\bar{\Lambda}(h)^{-1} \bar{Q} (e_\theta(t) + \bar{Z}(h, t) - \bar{d}(h, t))$$

and by introducing the above equation in the system

equations:
$$\begin{cases} \dot{y}_1 = L_f y_1 + L_g y_1 u \\ \ddot{y}_2 = L_f^2 y_2 + L_g L_f y_2 u \end{cases}$$
, we obtain the

tracking errors dynamics:

$$\begin{cases} \dot{e}_1 + \frac{1}{h} e_1 = 0 \\ \ddot{e}_\theta + \frac{3}{h} \dot{e}_\theta + \frac{6}{h^2} e_\theta + \frac{6}{h^3} e_\theta = 0 \end{cases}$$

Here also the above dynamics equations are linear and time invariant. We can show that, by using Routh criterion, the above equations are asymptotically stable, therefore, the augmented system closed by the control given in equation (12) is stable.

4. Simulation results

Computer simulations have been performed to check the behavior of the proposed control scheme. The plant under control is PMSM used in [11] with the following parameters:

$$R = 17.201 \Omega; \quad p = 4; \quad L_q = L_d = L = 0.038136 H;$$

$$\Phi = 0.052325 Wb; \quad B = 0.0025 Nms; \quad J = 0.001197 kgm^2$$

The parameter values of the smoothed reference signals are chosen as follows:

$$y_{ref1} = 0, y_{ref2} = 50 \text{ rad/s}, \xi = 1 \text{ and } \omega = 15.$$

To examine the speed tracking performances, it was considered that the speed must reach the value: $\Omega = 50 \text{ rad/s}$ in the interval of time simulation 0-3s. To test the disturbance rejection, a 1.5Nm unknown load torque is applied in the interval 0.8-2s and it is decreased to 1Nm for $t > 2s$. After several trials, the optimal control parameters are chosen as:

$$h = 0.01, \quad Q = 10^3 I_2, \quad R = 10^{-2} I_2$$

The first control algorithm without integral action (7) has been applied to PMSM. Figure 1 shows the behavior of the currents i_d, i_q and the speed $\Omega(t)$. In the unload case, i.e from $t = 0$ to $t = 0.8s$, it appears that the motor speed track closely the reference trajectory with a short steady state error. However, when the load torque is applied (between $t = 0.8s$ and $t = 2s$) the steady state error has increased. Figure 2 depicts the variation of the voltages (u_d, u_q), speed tracking error and the applied load torque.

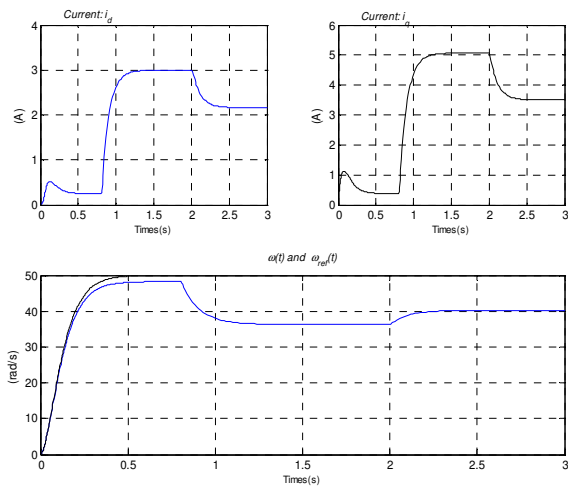


Fig 1. Tracking performance in matched case.

As it is stated in the introduction, the variation of parameters during operation degrades the tracking performances. To check the robustness of the proposed algorithm, the mismatched case consists of:

- Variation of resistance parameter R as it is shown in figure 4 which covers the more realistic situation,
- The no vanishing load torque (figure 4).

In mismatched case, the second control algorithm with integral action (12) has been applied to the same PMSM. The resulting d -axis current, q -axis current and the speed tracking performance are shown in figure 3. The behavior of the speed $\Omega(t)$ is very close to the reference speed $\Omega_{ref}(t)$ and it is clear that neither resistance variation nor the applied load torque have an effect on the motor speed tracking performances since it has been rapidly compensated (fig.4).

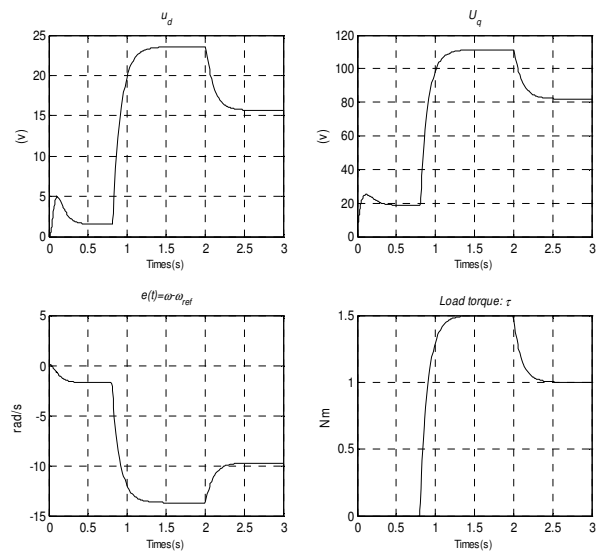


Fig 2. Control signal (voltages), speed tracking error and smoothed load torque.

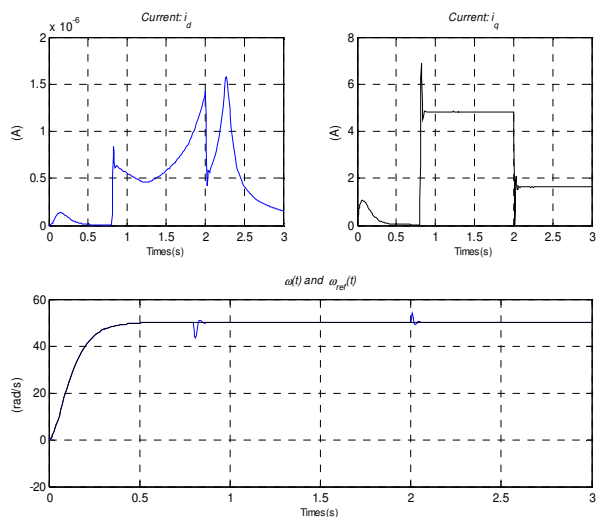


Fig 3. Tracking performance in mismatched case.

5. Conclusion

In this paper, it has been shown that using one-step-ahead predictive controller is effective for solving the speed tracking problem of PMSM. To enhance the robustness of the controller with respect to unknown load torque an integral action is incorporated in the loop. Indeed, simulations show an accurate tracking of speed reference signal with faster load torque disturbance rejection. Thus, the load torque observer is not needed in this control scheme.

Additional research should be oriented first towards the sensorless control algorithm where the mechanical sensor for motor speed is not needed and great advantages (cost function and reliability improvement) are guaranteed. Second towards the discrete implantation of the continuous-time nonlinear predictive controller.

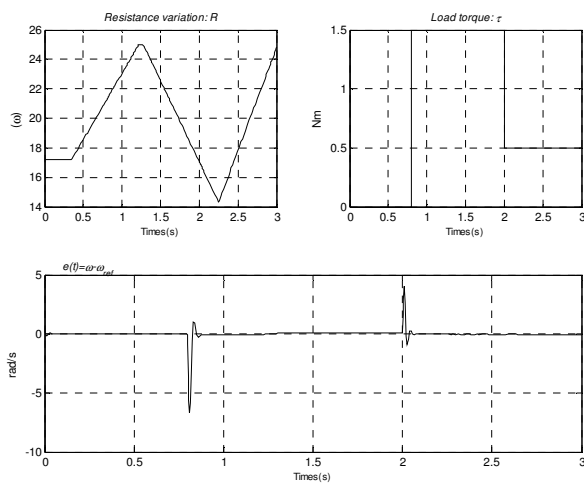


Fig 4. Resistance variation, load torque and speed tracking performance in mismatched case.

Appendix:

The Lie-derivatives of the output are:

$$L_f y_1 = -\frac{R}{L_d} x_1 + p \frac{L_q}{L_d} x_2 x_3; \quad L_g y_1 = \begin{bmatrix} \frac{1}{L_d} & 0 \end{bmatrix};$$

$$L_g L_f y_2 = \begin{bmatrix} 0 & \frac{3 p \Phi}{2 J L_q} \end{bmatrix}; \quad L_f y_2 = -\frac{B}{J} x_3 + \frac{3 p \Phi}{2 J} x_2;$$

$$L_f^2 y_2 = -\frac{3 p}{2 J} \Phi \left(\frac{R}{L_q} + \frac{B}{J} \right) x_2 - \left(\frac{3 p^2 \Phi^2}{2 J L_q} - \frac{B^2}{J^2} \right) x_3 - \frac{3 p^2 \Phi L_d}{2 J L_q} x_1 x_3$$

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