Cyclic Cancellation Coding for OFDM Over a Multi-path Rayleigh Fading Channels

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Abstract—**Inter-carrier-interference (ICI) in an orthogonal frequency division multiplexing (OFDM) system stems from two factors, the frequency offset between the transmitter and receiver local oscillators and the time variation of the mobile channel. This paper introduce an efficient ICI cancellation coding scheme called cyclic cancellation coding (CCC). CCC is the same as the first-order polynomial cancellation coding (PCC) [14] and symmetric cancellation coding (SCC) [16], [17] except only that each data symbol is modulated onto sub-carriers at the cyclic positions rather than adjacent sub-carriers as in PCC and symmetric or mirror image positions as in SCC. The main feature of the new scheme is that it exploit the frequency diversity better than SCC. We study the effectiveness of CCC for time-variant Rayleigh fading channels and compare its performance with that of PCC and SCC. Our results show that, the scheme provide excellent results compared to PCC and SCC.**

I. INTRODUCTION

Recently orthogonal frequency division multiplexing (OFDM) has been considered for various types of digital communications such as digital video broadcasting (DVB) [1], digital audio broadcasting (DAB) [2], and mobile integrated service digital network (ISDN)[3] (Also see [4].) In an OFDM communication system, the channel is divided into mutually orthogonal sub-channels, and information symbols are transmitted in the form of long waveforms over the sub-channels in parallel. The long symbol duration makes the system immune to multi-path. However, frequency offset between the transmitter and receiver local oscillators or time variation of the channel destroys the orthogonality among sub-channels and causes inter-carrier interference (ICI) [4], [5], [8], [12]. Besides the efforts to mitigate the effect of frequency offset based on its direct estimation [12], several schemes which reduce the effect of ICI have been proposed. In [10], [11], and [12], frequency-domain equalizers are used to compensate for ICI. Time-domain windowing [13] is another way to deal with ICI. Recently, however, polynomial cancellation coding (PCC) [13], [14], [15] and symmetric cancellation coding (SCC) [16], [17], [18] have been shown to be a good solution for ICI reduction in OFDM systems . The main ideas of the two schemes are same, however, in PCC, one data symbol is modulated onto adjacent sub-carriers with predefined weighting coefficients while in SCC the data symbol is modulated onto sub-carriers at the mirror image positions instead of adjacent positions. It is shown in [16], [17] and [18] that the SCC outperforms the PCC due to the frequency diversity gained by the scheme.

The frequency diversity effect gained by SCC comes from the difference in frequency range between the two sub-carriers carrying the same data symbol. And, since the locations of those two sub-carriers are symmetric, the sub-carriers which are located around the center of the OFDM frequency range will lose such advantages due to less difference between the sub-carrier frequencies.

In this paper we introduce another ICI cancellation scheme called cyclic cancellation coding (CCC). The main idea of CCC is the same as PCC and SCC except that one data symbol is modulated onto sub-carriers at the cyclic positions instead of adjacent positions in PCC and symmetric or mirror image position in SCC. In this scheme, the diversity effect is maintained over the all sub-carrier pairs. We show that the new scheme has better performance than PCC and SCC over time-variant frequency selective Rayleigh fading channels. Also, we show that the estimation method which is proposed my Moose in [12] is applicable in the OFDM system employing CCC (CCC-OFDM).

II. NEW CANCELLATION CODING SCHEME

A. System Model

The discrete-time baseband OFDM system we consider is shown in Fig. 1. The input bit stream is mapped to a series of complex-valued data symbols by the symbol modulator. These data symbols are arranged to parallel data symbols $a'(n), 0 \leq n \leq N'-1$, and encoded by the CCC encoder. Let $a(n), 0 \leq n \leq N-1$, be the CCC encoder output for $N' = N/2$. The parallel data $a(n)$ are then modulated by N-point inverse discrete-time Fourier transform (IDFT) onto N sub-carriers and cyclic prefix (CP) is appended as a guard symbol. Then, the discrete-time representation of one OFDM block is given by

$$x(k) = \sum_{n=0}^{N-1} a(n) e^{j2\pi nk/N}, \quad -G \leq k \leq N-1, \quad (1)$$

where G is the length of the guard interval. We let T (sec) be the length of an OFDM symbol without guard symbols

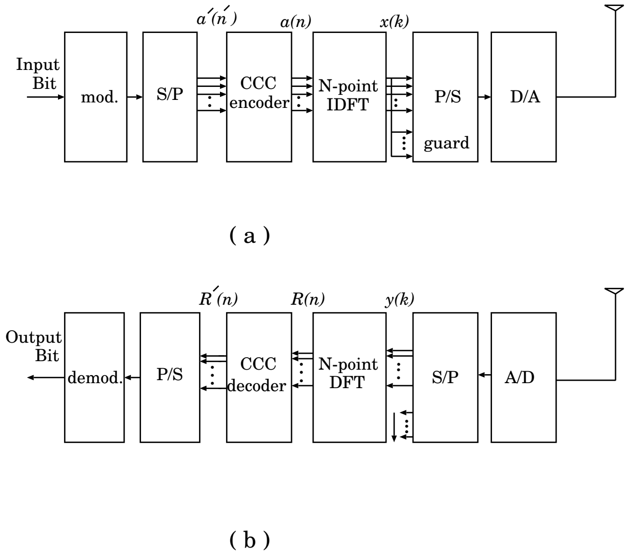


Fig. 1. A block diagram of the CCC-OFDM system, (a) transmitter, (b) receiver

and let $T_s = T/N$ be the sampling period assumed in the OFDM system [4]. The Overall length (in seconds) of an OFDM symbol is thus $\hat{T} = (N+G)T_s$.

We assume an L-paths Rayleigh fading channel with Doppler spread,

$$y(k) = \sum_{l=0}^{L-1} h(k,l)x(k-l) + w(k), \qquad (2)$$

where $h(k,l)$ is the complex amplitude of the lth path at time k and $w(k)$ denotes a complex additive white Gaussian noise (AWGN) with mean zero and variance $N{\cdot}N_0$. This channel model includes constant frequency offset channels as well. We suppose $G \geq L-1$ in the following discussions.

At the receiver, after the guard interval was removed, demodulation is performed by discrete Fourier transform (DFT) as

$$\begin{aligned} R(n) &= \frac{1}{N}\sum_{k=0}^{N-1} y(k)e^{-j2\pi nk/N} \\ &= \sum_{m=0}^{N-1}\sum_{l=0}^{L-1} a(m)H_l^{(n-m)}e^{-j2\pi lm/N} + W(n), \end{aligned} \qquad (3)$$

where $W(n)$ is the DFT of the AWGN $w(k)$, which is again a white Gaussian noise, and $H_l^{(n)}$ represents the DFT of $h(k,l)$ in k,

$$H_l^{(n)} = \frac{1}{N}\sum_{k=0}^{N-1} h(k,l)e^{-j2\pi kn/N}. \qquad (4)$$

Now, let $A(m,n) = \sum_{l=0}^{L-1} H_l^{(n-m)}e^{-j2\pi lm/N}$.

Then, we can rewrite $R(n)$ in (3) as, for $0 \leq n \leq N-1$,

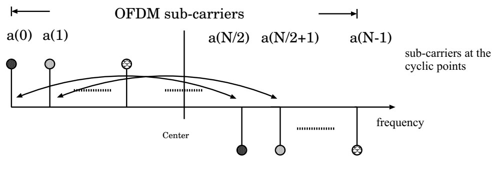
$$R(n) = A(n)a(n) + I(n) + W(n), \qquad (5)$$

where $A(n) = A(n,n)$ is the multiplicative distortion of the nth sub-carrier and $I(n)$ is the ICI term given by

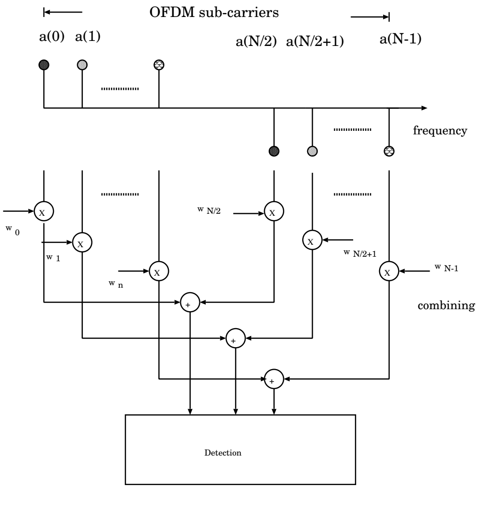
$$I(n) = \sum_{\substack{m=0 \\ m\neq n}}^{N-1} a(m)A(m,n). \qquad (6)$$

B. Cyclic Cancellation Coding (CCC)

Cyclic Cancellation Coding (CCC) is very simple, Fig. 2. Its idea is to modulate one data symbol onto two sub-carriers at the cyclic positions with a 180° phase shift between them. Suppose that N sub-carriers of the OFDM system are indexed from 0 to $N-1$. Then, $a(0)$ and $a(\frac{N}{2})$, $a(1)$ and $a(\frac{N}{2}+1)$, and so on are pairs of sub-carrier values at the cyclic positions and, given information symbols $a'(n)$, $0 \leq n \leq N/2$, they are determined as $a(0) = -a(\frac{N}{2}) = a'(0)$, $a(1) = -a(\frac{N}{2}+1) = a'(1)$, and so on. We note that, in the first-order PCC considered in [14], [15], each data symbol is modulated over adjacent sub-carriers as $a(0) = -a(1) = a'(0)$, $a(2) = -a(3) = a'(1)$, and so on. On the other hand, in SCC [16], [17], [18], each data symbol is modulated onto sub-carriers at the mirror image positions as $a(0) = -a(N-1) = a'(0)$, $a(1) = -a(N-2) = a'(1)$ and so on. The information symbols $a'(n)$ are assumed to be independent and identically distributed with mean zero and variance E_s.



(a) coding



(b) decoding

Fig. 2. Cyclic cancellation coding, (a) coding, (b) decoding

C. Combining for the CCC-OFDM

Combining is essential for the OFDM system employing SCC and CCC. It is shown in [18] that the subtraction combining in SCC-OFDM is effective in the case of flat Rayleigh fading channels and frequency offset channels. However, this simple combining does not allow SCC-OFDM to realize frequency diversity for frequency-selective fading, which is the merit considered in [16], [17] and this merit is maintained by using differential modulation for a certain level of Doppler spread as shown in [18]. It is shown in [16], [17], [18], that the use of maximal ratio combining (MRC) allow SCC to realize frequency diversity and to outperform PCC over frequency selective Rayleigh fading channels [16], [17], [18].

In the receiver, we consider the same combining as SCC, $R'(n) = R(n) - R(\frac{N}{2} + n)$, between two sub-carriers at the cyclic positions. Here, it is important to mention that, although we are using the subtraction combining with the diversity combining, the effect of diversity combining can be seen by simulation results only.

We have, for $0 \leq n \leq N/2 - 1$,

$$\begin{aligned} R'(n) &= R(n) - R(\frac{N}{2} + n) \\ &= \sum_{m=0}^{\frac{N}{2}-1} \sum_{l=0}^{L-1} a(m) \left[H_l^{(n-m)} - H_l^{(n-m+\frac{N}{2})} e^{-\pi l}\right] \\ &\quad \times e^{-j2\pi lm/N} + W(n) \\ &= a'(n)A'(n) + I'(n) + W'(n), \end{aligned} \tag{7}$$

where we let $W'(n) = W(n) - W(\frac{N}{2} + n)$, $A'(n) = A'(n,n)$, and

$$I'(n) = \sum_{\substack{m=0 \\ m \neq n}}^{\frac{N}{2}-1} a'(m)A'(m,n) \tag{8}$$

for

$$\begin{aligned} A'(m,n) &= \sum_{l=0}^{L-1} a(m) \left[-H_l^{(n-m-\frac{N}{2})} e^{-\pi l}\right. \\ &\quad \left. +2H_l^{(n-m)} - H_l^{(n-m+\frac{N}{2})} e^{-\pi l}\right] e^{-j2\pi lm/N}, \end{aligned} \tag{9}$$

D. Detection

For the detection of the transmitted data, we assume that the multiplicative distortions $A(n)$ in (5) and $A'(n)$ in (7) are perfectly estimated. Then $\frac{R(n)}{A(n)}$ and $\frac{R'(n)}{A'(n)}$ give decision values for detecting data symbols for the normal OFDM and for the CCC-OFDM, respectively. For the PCC-OFDM with the first-order PCC, and SCC-OFDM, we note, the decision value is $\frac{R''(n)}{A''(n)}$ where $R''(n) = R(2n) - R(2n+1)$ and $R''(n) = R(n) - R(N-n-1)$, respectively [15],[18].

E. Bit rate normalization

Since the number of sub-carriers available for data is halved for the same N in PCC, SCC and CCC, the bit error rate (BER) of the normal OFDM with BPSK or 4QAM should be compared with that of SCC, PCC (or CCC) with QPSK or 16QAM.

III. SINR ANALYSIS

Suppose that the channel is a Rayleigh fading channel with wide sense stationary uncorrelated scattering (WSSUS) [21] and hence that $h(k,l)$ satisfies

$$E\left[h^*(k,l)h(k',l')\right] = J_0(2\pi f_D T_s[k-k'])p_l \delta(l-l') \tag{10}$$

where p_l is the normalized power-delay profile of the channel such that $p_l \geq 0$ and $\sum_{l=0}^{L-1} p_l = 1$, $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind, f_D is the Doppler frequency, and $\delta(l)$ is Kronecker's delta function.

We employ the same approach as those used in [7], [15], and [19], which are employed from [6], and assume that $I'(n)$ is a mutually independent Gaussian random variable independent of $H'(n)$. Then, the signal-to-interference plus noise power ratio (SINR)[1] is given by

$$\Gamma = \frac{E[|H(n)|^2]}{E_s^{-1}E[|I(n)|^2] + \Gamma_0^{-1}}. \tag{11}$$

where $\Gamma_0 = E_s/N_0$ is the signal-to-noise power ratio (SNR).

Then, the SINR for the PCC-OFDM is given as [15][2]

$$\Gamma = \frac{\frac{4}{N^2}\sum_{k,k'} \alpha(k-k')\beta(k)\beta(k')}{3 + \alpha(\frac{N}{2}) - \frac{4}{N^2}\sum_{k,k'} \alpha(k-k')\beta(k)\beta(k') + 2\Gamma_0^{-1}}, \tag{12}$$

and the SINR of SCC-OFDM,is given as [18]

$$\Gamma(n) = \frac{B(n,n)}{\sum_{\substack{m=0 \\ m \neq n}}^{\frac{N}{2}-1} B(m,n) + 2\Gamma_0^{-1}}, \tag{13}$$

where $B(m,n)$ is given as

$$\begin{aligned} B(m,n) &= \frac{16}{N^2} \sum_{l=0}^{L-1} \sum_{k,k'} \alpha(k-k') p_l \sin\frac{2\pi k(n+\frac{1}{2})}{N} \\ &\quad \times \sin\frac{2\pi k'(n+\frac{1}{2})}{N} \sin\frac{2\pi(k-l)(m+\frac{1}{2})}{N} \\ &\quad \times \sin\frac{2\pi(k'-l)(m+\frac{1}{2})}{N} \end{aligned} \tag{14}$$

For the CCC-OFDM, the SINR is calculated as

$$\Gamma = \frac{\frac{4}{N^2}\sum_{k,k'} \alpha(k-k')\gamma(k)\gamma(k')}{4 - \frac{4}{N^2}\sum_{k,k'} \alpha(k-k')\gamma(k)\gamma(k') + 2\Gamma_0^{-1}}, \tag{15}$$

where we let $\sum_{k,k'} = \sum_{k=0}^{N-1}\sum_{k'=0}^{N-1}$ and let

$$\begin{aligned} \alpha(k) &= J_0(2\pi f_D T_s k) \\ \beta(k) &= 1 - \cos\left(\frac{2\pi k}{N}\right) \\ \gamma(k) &= 1 - \cos(\pi k) \end{aligned} \tag{16}$$

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[1] If $N_0 = 0$, it is the signal-to-interference power ratio (SIR).

[2] The factor 2 before Γ_0^{-1} is erroneously omitted in [15].

Fig. 3 compares the SIR (Γ with N_0=0) between the normal OFDM, PCC-OFDM, SCC-OFDM, and CCC-OFDM, with N=1024 over the multipath Raleigh fading. In the figure, we compare the SIR of the decision value at $n = \frac{N}{2} - 1$ for the normal OFDM and the decision values at $n = \frac{N}{4} - 1$ for the PCC-OFDM, SCC-OFDM, and CCC-OFDM, respectively. We consider four-path fading channels given by exponential power delay profile with delay spread T_s. Although the diversity effect was not included in our analysis, in the figure, it is clear that CCC-OFDM has better performance than the SCC-OFDM and the normal OFDM. However, the effect of diversity will enhance the performance of both, the SCC-OFDM and the CCC-OFDM, and their performance will overcomes the PCC-OFDM as we will see in the next section.

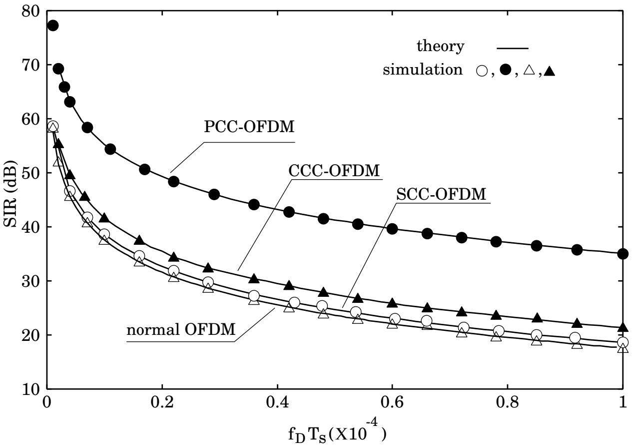

Fig. 3. SIR comparison between the normal OFDM, PCC-OFDM, SCC-OFDM, and CCC-OFDM over multipath fading with exponential power delay channel, N=1024.

A. BER Comparison

We observed that the bit-rate of the PCC-OFDM, SCC-OFDM, and CCC-OFDM is one half of that of the normal OFDM for the same modulation level, and hence higher modulation levels should be used for the PCC-OFDM, SCC-OFDM, and CC-OFDM to realize the same bit-rate as shown in Table I.

TABLE I
MODULATION LEVELS OF SCC-OFDM AND PCC-OFDM RELATED TO NORMAL OFDM.

Schemes	symbol modulation		
	low	medium	high
OFDM	BPSK	4QAM	16QAM
CCC-OFDM (PCC-OFDM) (SCC-OFDM)	4QAM	16QAM	256QAM

In this section, we consider the BER performance of the normal OFDM, PCC-OFDM, SCC-OFDM, and CCC-OFDM over a time-varying multi-path fading channel, respectively. The channel is four paths with exponential power delay profile. The power of each path is attenuated by 1 dB. We assume perfect channel estimation and apply the decision scheme in Section 2.3. We assume the systems parameters shown in Table II.

TABLE II
SYSTEMS PARAMETERS.

Total bandwidth	1 MHz
Bite rate	888.8 Kb/s
Carrier frequency	2GHz
Channel	Rayleigh with exponential delay profile
Symbol period	128 μsec.
Guard symbols	16
Sub-carrier number N	128
Paths L	4
Sub-carrier spacing	7.8 KHz
Channel estimation	perfect

We next consider the BER vs. SNR performance. In this simulation, we concentrate on $f_D T_s = 0.00037$. This value give $f_D T$=0.047 for $N = 128$. In Fig. 4 we show the results for the normal OFDM, PCC-OFDM, SCC-OFDM and CCC-OFDM. In the figure, we notice that, the SCC-OFDM and CCC-OFDM outperform the normal OFDM and the PCC-OFDM due to the diversity gain with those two schemes. However, the performance of the CCC-OFDM started to overcomes that of SCC-OFDM due to higher diversity gain compared to the SCC-OFDM.

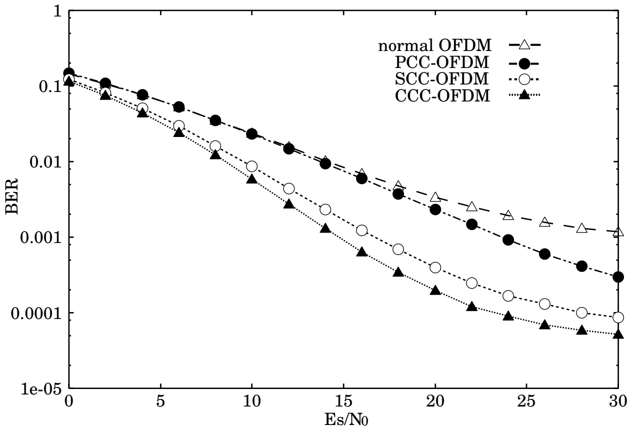

Fig. 4. BER versus E_s/N_0 for the normal OFDM (BPSK), PCC-OFDM (4-QAM), SCC-OFDM (4-QAM), and CCC-OFDM $f_D T$=0.0474, N=128.

IV. CHANNEL ESTIMATION

Channel estimation is one of the main problems in OFDM system, especially in the time-variant channels. Moose in [12] proposed a scheme to estimate the channel. Comparing the methodology of CCC=OFDM, it is interesting to see that such method is applicable for CCC-OFDM. And applied the method to CCC-OFDM is an interesting point.

V. CONCLUSIONS

This paper proposed a new ICI cancellation coding, called cyclic cancellation coding (SCC), which reduces the impact of ICI on OFDM systems. Under the condition of the same bandwidth efficiency we compared the proposed CCC-OFDM system with the normal OFDM and OFDM system using PCC and SCC(PCC-OFDM, SCC-OFDM) [14][16] , and we confirmed that the proposed CCC-OFDM system performs much better than the normal OFDM, PCC-OFDM and SCC-OFDM systems due to the diversity gain gained by this scheme. Therefore, we can conclude that the proposed scheme is very effective to reduce the effect of ICI for time-varying frequency selective fading channels.

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